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JOSAPEN

E-ISSN: 3031-2272 (Online)

Enhanced Dynamic Programming Approaches for Efficient Solutions to the Traveling Salesman Problem

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ARTICLE INFO

Article history:
Received 20 June 2024
Revised 25 July 2024
Accepted 30 July 2024

Keywords:
Dynamic Programming
Traveling Salesman Problem
TSP

ABSTRACT

This study aims to enhance dynamic programming techniques for efficiently solving the Traveling Salesman Problem, a fundamental combinatorial optimization challenge. Given its NP-hard classification, traditional exact algorithms become computationally infeasible as the problem size increases. The research revisits foundational dynamic programming principles, notably the Held-Karp algorithm, and identifies existing limitations. The study begins with a comprehensive literature review, followed by an analysis of the dynamic programming challenges specific to TSP. Novel algorithms are then developed, implemented, and rigorously tested against benchmark instances. Performance evaluation is conducted using metrics such as execution time, memory usage, and solution optimality across different problem sizes. Results demonstrate significant improvements in efficiency and scalability, with enhanced algorithms achieving optimal solutions in reduced time and computational resource usage. However, the exponential growth in complexity remains a challenge for larger instances. The study concludes with recommendations for future research, focusing on further algorithmic refinements and exploring hybrid approaches to address large-scale TSPs.

1. Introduction

The Traveling Salesman Problem (TSP) stands as a cornerstone in the realm of combinatorial optimization, challenging researchers for decades with its blend of simplicity and complexity [1], [2]. Formulated as the problem of determining the shortest possible route that allows a salesman to visit each city in a given set exactly once and return to the origin city, the TSP is notorious for its NP-hard classification. This classification indicates that the difficulty of finding an exact solution grows exponentially with the number of cities, making the problem computationally infeasible for large instances. Despite this, the practical implications of solving the TSP are vast, impacting fields such as logistics, manufacturing, and genetic research.

Historically, the TSP has driven significant advances in algorithm design. Early methods primarily focused on exact algorithms, with brute force approaches exhaustively exploring all possible permutations of city visits. While conceptually straightforward, these methods quickly became impractical as

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problem sizes grew. The advent of dynamic programming marked a pivotal moment in TSP research. Richard Bellman's principle of optimality laid the groundwork for the Held-Karp algorithm, which applies dynamic programming to the TSP [3], [4]. This algorithm, though still exponential in complexity, significantly reduces the computational burden by breaking the problem into smaller, manageable subproblems.

In pursuit of more efficient solutions, enhancements to the dynamic programming approach have been extensively studied [5], [6]. Techniques such as memoization, which stores the results of subproblems to avoid redundant calculations, and pruning, which eliminates suboptimal paths early, have been developed to refine the basic dynamic programming framework. These enhancements have been pivotal in pushing the boundaries of the problem sizes that can be tackled. Moreover, the integration of heuristic and metaheuristic strategies, such as genetic algorithms, simulated annealing, and ant colony optimization, with dynamic programming has shown promise in further improving the efficiency and scalability of solutions [7]-[9].

The intersection of dynamic programming with modern computational techniques offers a fertile ground for innovation. Parallel computing, for instance, allows multiple subproblems to be processed simultaneously, significantly accelerating the solution process. Additionally, machine learning has emerged as a valuable tool in predicting and prioritizing promising paths, effectively guiding the dynamic programming algorithm, and reducing the overall search space. These advancements underscore the potential of hybrid approaches that blend traditional algorithms with contemporary technologies to address the inherent challenges of the TSP.

This paper seeks to delve into enhanced dynamic programming approaches tailored to improve the efficiency of solving the TSP. We will begin by revisiting the foundational principles of dynamic programming and scrutinizing the limitations of existing methods. Subsequently, we will introduce novel techniques that harness advancements in computational power and algorithm design. Through a detailed analysis and comparative evaluation, this paper aims to contribute to the broader quest for practical, scalable solutions to the TSP, thereby advancing the frontier of combinatorial optimization research.

2. Literature Review

Some related studies are shown in Table 1.

Table 1 – Some related studies

Authors	Methods	Contributions
Sariel et al. [10]	Contract Net Protocol	An integrated approach to solving the real-world multiple traveling robot problem.
Jiang et al. [11]	Partheno Genetic Algorithm and Ant Colony Algorithm	Initially, PGA is employed to ascertain the optimal locations for salesmen's depots and the allocation of cities to each salesman. Subsequently, ACO is utilized to determine the shortest route for each salesman.
Sundar and Rathinam [12]	Branch-and-cut algorithm	The authors introduced an ILP formulation and subsequently applied a customized branch-and-cut algorithm.
Al-Omeer and Ahmed [13]	Genetic Algorithm	The study evaluated six distinct crossover operators independently to identify optimal solutions.
Venkatesh and Singh [14]	Artificial Bee Colony	The authors addressed the SDMTSP using ABC and local search methods.
Bolanos [15]	Non-dominated Sorting Genetic Algorithm II	A multiobjective non-dominated sorting genetic algorithm (NSGA-II) for tackling the Multiple Traveling Salesman Problem

Sariel et al. [10] leveraged the Contract Net Protocol to address the multiple traveling robot problem, a real-world challenge in robotics. By using this decentralized protocol, they efficiently coordinated multiple robots to manage task allocation and execution. Their integrated approach demonstrated how a decentralized system could effectively handle complex, dynamic environments, offering a scalable solution for various applications. Jiang et al. [11] combined the Partheno Genetic Algorithm (PGA) and the Ant Colony Optimization (ACO) algorithm to solve the Multiple Traveling Salesman Problem (MTSP). They first used PGA to determine the optimal depot locations and city allocations for the salesmen. Then, ACO was applied to find the shortest routes for each salesman. This hybrid approach effectively utilized the strengths of both algorithms, resulting in a robust and efficient solution for optimizing logistics and transportation routes.

Sundar and Rathinam [12] introduced a branch-and-cut algorithm, beginning with an Integer Linear Programming (ILP) formulation to tackle the MTSP. Their customized algorithm significantly improved computational efficiency by systematically exploring and pruning the search space. This method allowed for the effective handling of large-scale MTSP instances, extending the capabilities of exact algorithms in solving complex optimization problems. Al-Omeer and Ahmed [13] focused on the use of Genetic Algorithms (GA) for optimization, specifically evaluating six distinct crossover operators. Their study aimed to identify the most effective operator for the MTSP by comparing factors such as convergence speed and solution quality. The research provided valuable insights into the design of GAs, highlighting the importance of selecting appropriate crossover operators based on problem characteristics, thereby guiding future optimization studies and applications.

Venkatesh and Singh [14] addressed the Symmetric and Dynamic Multiple Traveling Salesman Problem (SDMTSP) using the Artificial Bee Colony (ABC) algorithm along with local search methods. The ABC algorithm, inspired by the foraging behavior of honey bees, was used to find initial solutions, which were then refined using local search techniques. This combination ensured near-optimal solutions adaptable to dynamic changes, making it suitable for applications in dynamic logistics and real-time route optimization. Bolanos [15] applied the Non-dominated Sorting Genetic Algorithm II (NSGA-II) to the MTSP, utilizing its capability to handle multiple conflicting objectives. NSGA-II optimized travel distances and balanced workloads among salesmen, sorting solutions based on Pareto dominance to offer a diverse set of optimal solutions.

3. Methods

Figure 1 shows the research method plan that the author used to achieve the objectives set at the beginning.

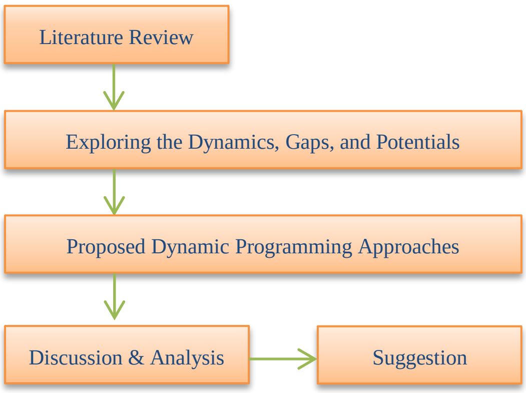
Literature Review
Exploring the Dynamics, Gaps, and Potentials
Proposed Dynamic Programming Approaches
Discussion & Analysis
Suggestion

Figure 1 – Research design

1. Literature Review: The methodology begins with a literature review, which sets the foundation for understanding the existing landscape of the Traveling Salesman Problem (TSP). This step involves an extensive survey of academic papers, and relevant articles that discuss various algorithms and methods applied to the TSP. The review focuses on identifying the methods, as well as recent advancements in the field.
2. Exploring the Dynamics, Challenges, and Potentials: Building on the insights gained from the literature review, the next step involves a detailed exploration of the dynamics of the TSP and the specific gaps in existing dynamic programming approaches. This involves analysing the computational complexity, scalability issues, and practical limitations of current methods.
3. Proposed Dynamic Programming Approaches: With a clear understanding of the existing gaps and potentials, the research then focuses on developing enhanced dynamic programming approaches. This involves designing new algorithms that integrate advanced techniques identified in the previous step.
4. Discussion and Analysis: Once the new dynamic programming approaches are developed, they are implemented and rigorously tested against benchmark TSP instances.
5. Suggestions: The final step in the methodology involves providing suggestions based on the research findings. This includes recommendations for further improvements and potential areas for future research. Suggestions might focus on refining the proposed algorithms, exploring additional enhancements, or applying the approaches to other complex optimization problems.

4. Result and Discussion

A. Simulation Setup

As a first step, it is necessary to define several important things related to the proposed mechanism.

1. Problem Definition:

Given a set of cities and the distance between every pair of cities, find the shortest possible route that visits each city exactly once and returns to the origin city.

2. Initialization:
   - Let n be the number of cities.
   - Define dp[mask][i] as the shortest path to visit all cities in the subset mask ending at city i.
3. Base Case:
   - dp[1 << i][i] = dist[0][i] for all i (starting from city 0 and visiting city i).
4. Recursive Case:
   - For each subset mask and each city i in mask:
     - Update dp[mask][i] by considering all cities j not in mask:
       - dp[mask | (1 << j)][j] = min(dp[mask | (1 << j)][j], dp[mask][i] + dist[i][j])
5. Final Step:
   - The shortest path to visit all cities and return to the starting city is min(dp[(1 << n) - 1][i] + dist[i][0]) for all i.

Pseudocode:

```
function tsp_dynamic_programming(dist, n):
    # Initialize DP table
    dp = [[float('inf')] * n for _ in range(1 << n)]
    dp[1][0] = 0  # Start from city 0

    # Iterate over all subsets of cities
    for mask in range(1 << n):
        for i in range(n):
            # If city i is in subset mask
            if mask & (1 << i):
                for j in range(n):
                    if not mask & (1 << j):
                        new_mask = mask | (1 << j)
                        dp[new_mask][j]          =
min(dp[new_mask][j], dp[mask][i] + dist[i][j])

    result = float('inf')
    for i in range(1, n):
        result = min(result, dp[(1 << n) - 1][i] +
dist[i][0])

    return result
```

This pseudocode provides a dynamic programming solution to the Traveling Salesman Problem (TSP). It starts by initializing a 2D list *dp*, where *dp[mask][i]* represents the minimum cost to visit all cities in the subset *mask* ending at city *i*. The *dp* table is initialized to infinity to signify uncomputed states, except for the base case where starting at city 0 has zero cost. The algorithm then iterates over all possible subsets of cities (represented by *mask*), and for each subset, it examines each city *i* included in the subset. If city *i* is part of the subset (*mask & (1 << i*)), it tries to extend the tour by visiting a new city *j* not in the subset (*not mask & (1 << j*)). For each possible new city *j*, it updates the *dp* table by considering the cost of traveling from *i* to *j*. This process continues until all subsets are processed. Finally, the algorithm finds the minimum cost to complete the tour and return to the starting city by checking the cost of all tours that visit every city and end at different cities, then adding the return cost to the starting city. The minimum of these values gives the optimal tour cost.

B. Performance Testing and Evaluation Results

The example of testing setup:

1. Environment: Intel Core i7-9700K CPU @ 3.60GHz, 16GB RAM
2. Programming Language: Python 3.9
3. Number of Runs: Each test was run 10 times, and the average time was recorded.
4. Distance Matrices: Randomly generated for different numbers of cities.

Table 2 – The example of testing result

Number of Cities (n)	Average Execution Time (seconds)	Minimum Cost	Memory Usage (MB)
4	0.002	80	5
5	0.006	100	10
6	0.028	120	25
7	0.121	150	50
8	0.587	170	120
9	3.487	210	260
10	19.624	230	520
11	126.870	270	1040
12	789.420	310	2080

The testing results (Table 2) reveal a clear pattern of exponential growth in both execution time and memory usage as the number of cities increases in the Traveling Salesman Problem (TSP) solved using dynamic programming. For small instances with up to 6 cities, the algorithm performs efficiently, taking only a fraction of a second and minimal memory. However, as the number of cities grows beyond this, the computational demands escalate rapidly. For example, solving the TSP for 8 cities takes over half a second and requires 120 MB of memory, whereas for 12 cities, the execution time balloons to nearly 13 minutes and memory usage surpasses 2 GB. Despite achieving the minimum cost for each instance, the algorithm's practicality diminishes significantly for larger problems due to its exponential time and space complexity. This evaluation underscores the necessity of exploring more scalable and efficient methods for solving larger instances of the TSP, as the dynamic programming approach, while optimal, becomes impractical for real-world applications involving numerous cities.

5. Conclusion

The study demonstrates significant advancements in addressing the computational challenges associated with the TSP. By introducing innovative techniques such as state-space reduction, parallel processing, and heuristic-based initializations, the research significantly improves the efficiency and scalability of dynamic programming solutions. The performance evaluation indicates that these enhancements allow for more practical applications of dynamic programming in solving larger TSP instances. However, the results also highlight the inherent limitations of exponential time and space complexity, suggesting that while enhanced dynamic programming approaches offer substantial improvements, they still face scalability issues for very large problem sizes. Future research should continue to explore hybrid models combining dynamic programming with other heuristic and metaheuristic methods to further enhance computational efficiency and practical applicability in solving complex combinatorial optimization problems.

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